

June 2001

Model for Small neutrino masses at the TeV Scale

Salah NASRI^{a1} Sherif MOUSSA^{a,b2}

^a *Department of Physics, Syracuse University, Syracuse, NY
13244-1130, USA.*

^b *Department of Mathematics, Faculty of Science, Ain Shams
University, Cairo, 11556, Egypt.*

Abstract

We propose a model for neutrino mass generation in which no physics beyond a TeV is required. We extend the standard model by adding two charged singlet fields with lepton number two. Dirac neutrino masses $m_{\nu_D} \leq MeV$ are generated at the one loop level. Small left handed majorana neutrino masses can be generated via the seesaw mechanism with right handed neutrino masses M_R are of order TeV scale.

¹ Electronic address : snasri@phy.syr.edu

² Electronic address : sherif@phy.syr.edu

Solar and atmospheric neutrino oscillation experiments[1], give a strong evidence for the neutrino masses and mixings. In the minimal standard model, neutrinos are massless, due to the absence of the right handed neutrino and the conservation of lepton number. Non zero Dirac neutrino masses can be obtained by adding, singlet fermions (right handed neutrinos) to the SM. However, in this case one needs to fine tune the Yukawa couplings to be $\sim 10^{-12}$, which is quite unnatural.

An elegant and natural explanation of the smallness of the neutrino masses compared to the quarks and charged lepton masses is provided by the seesaw mechanism[2]. In this case the Majorana neutrino mass matrix is given by:

$$m_\nu = -m_D^T M_R^{-1} m_D, \quad (1)$$

where m_D is the Dirac neutrino mass matrix and M_R denotes the right handed Majorana neutrino mass matrix. This mechanism can be implemented naturally in many extensions of the standard model gauge group (e.g, in $SO(10)$ the right handed neutrino masses are associated with the $(B - L)$ symmetry breaking scale)[3]. The smallness of the neutrino mass is due to the suppression by the scale of the heavy fields (right handed neutrinos) after integrating them out. An equally attractive mechanism has been proposed recently [4] where the lepton number is broken explicitly by the coupling of a heavy higgs triplet to the standard model doublet.

Another interesting way to generate small Majorana neutrino masses is given by the Zee model[5], in which the masses are generated at the one loop level. In addition to the standard model higgs doublet Φ_{SM} the model contains a charged scalar field $h^{(+)}$, that is a singlet of $SU(2)_L$, and another doublet scalar field Φ which has the same quantum numbers as the standard model higgs. The singlet $h^{(+)}$ carries lepton number -2 , so that the total

lepton number L is conserved in the Yukawa sector. The lagrangian of the Zee model reads:

$$\mathcal{L}_{Zee} = f_{\alpha\beta} L_{\alpha}^T C i\tau_2 L_{\beta} h^{(+)} + \mu \Phi_{SM}^T i\tau_2 \Phi h^{(-)} + h.c., \quad (2)$$

where L_{α} is the lepton doublet, C denotes the charge conjugation, and $i\tau_2$ is the $SU(2)$ Levi-Civita symbol. Lepton number is broken explicitly by two unites throught the cubic coupling $\mu \Phi_{SM}^T i\tau_2 \Phi h^{(-)}$. In addition, a discrete symmetry is needed such that the higgs doublet Φ does not couple to leptons. A calculable neutrino Majorana masses are generated at one loop through the exchange of the physical higgs and charged lepton fields. Fermi statistics imply that the coupling matrix $f_{\alpha\beta}$ is antisymmetric, which leads to a neutrino mass matrix with vanishing diagonal elements.

The possibility of applying the radiative mechanism to generate Dirac fermion masses, including neutrinos, have been studied previously in the context of left right symmetric and superstring inspired models [6]. The authors of Ref. [7] have presented a model with large magnetic moment of the electron neutrino, by adding a singly charged higgs field , in addition to the right handed neutrinos. However, in this model the one loop neutrino Dirac mass is logarithmically divergent and must be cancelled by adding a counterterm, making the neutrino masses arbitrary.

In this letter we propose a model in which lepton number is broken by the right handed Majorana mass $M_R \sim TeV$, and where calculable Dirac neutrino masses are induced at one loop via the exchange of singlet charged scalar fields.

The model is the standard model extended by introducing two singlet charged scalar fields, S_1 and S_2 . No extra doublet is needed here. We also introduce three families of right-handed neutrinos $\nu_{R_1}, \nu_{R_2}, \nu_{R_3}$. Let us assume for the moment that the total lepton number is conserved. The most general relevant part of the lagrangian consistent with this

symmetry is given by:

$$\mathcal{L}_{ext} = f_{\alpha\beta} L_{\alpha}^T C i \tau_2 L_{\beta} S_1^{(+)} + g_{\alpha\beta} l_{R\alpha}^T C \nu_{R\beta} S_2^{(+)} + h.c \quad (3)$$

where, α, β denote generation indices. The couplings $f_{\alpha\beta}$ are antisymmetric in α and β , while $g_{\alpha\beta}$ are arbitrary. Conservation of lepton number requires that $S_1^{(+)}$ and $S_2^{(+)}$ carry lepton number $L = -2$. However at this stage a coupling between the standard model higgs and the right handed neutrino is allowed. The presence of such term in the lagrangian leads to a Dirac mass for the neutrino, and one needs to fine tune the Yukawa coupling to produce $m_{\nu} \ll m_e$. To forbid these couplings we impose a Z_2 symmetry acting as follows:

$$(L_{\alpha}, \Phi_{SM}, S_1^{(+)}) \longrightarrow (L_{\alpha}, \Phi_{SM}, S_1^{(+)}) \quad (4)$$

$$(N_{R\alpha}, S_2^{(+)}) \longrightarrow -(N_{R\alpha}, S_2^{(+)}) \quad (5)$$

In the limit where Z_2 is unbroken the Dirac mass for the neutrino vanishes to any order of perturbation theory. We will therefore assume that Z_2 is broken softly in the higgs sector.

The scalar potential is defined by:

$$V = V_1(\Phi_{SM} \Phi_{SM}^{\dagger}, S_1 S_1^{\dagger}, S_2 S_2^{\dagger}) + \kappa S_1 S_2^{\dagger} \quad (6)$$

where $V_1(\Phi_{SM} \Phi_{SM}^{\dagger}, S_1 S_1^{\dagger})$ contains the usual terms with positive mass square and quadratic terms for S_1 and S_2 such that $\langle S_1 \rangle = \langle S_2 \rangle = 0$ (since they carry electric charge). The term $\kappa S_1 S_2^{\dagger}$ is the Z_2 soft breaking term which can be of order of the charged singlet higgs masses m_S . The physical charged higgs fields are given in the basis $S_1^{(+)}$ and $S_2^{(+)}$ by:

$$\begin{aligned} h_1 &= S_1^{(+)} \cos \theta + S_2^{(+)} \sin \theta \\ h_2 &= -S_1^{(+)} \sin \theta + S_2^{(+)} \cos \theta \end{aligned} \quad (7)$$

where

$$\sin^2 2\theta = \frac{4\kappa^2}{4\kappa^2 + (m_2 - m_1)^2} \quad (8)$$

Dirac masses of the neutrino are generated from the one loop diagrams shown in Fig.1 and in the basis in which the charged lepton mass matrix is diagonal are given by

$$m_{\nu_{\alpha\beta}} = g_{\alpha\gamma} m_\gamma f_{\gamma\beta} \sin \theta \cos \theta I(M_1, M_2, m_\beta) \quad (9)$$

where m_β are the masses of the charged leptons, M_1, M_2 are the masses of the physical charged higgs h_1 and h_2 and $I(M_1, M_2, m_\beta)$ is the one loop integral given by:

$$I(M_1, M_2, m_\beta) = \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 - m_\beta^2} \left(\frac{1}{k^2 - M_2^2} - \frac{1}{k^2 - M_1^2} \right), \quad (10)$$

in evaluating this integral the charged lepton masses can be safely neglected so that the one loop induced Dirac neutrino masses are given by

$$m_{\nu_{\alpha\beta}} = \frac{1}{64\pi^2} g_{\alpha\gamma} m_\gamma f_{\gamma\beta} \sin(2\theta) \ln\left(\frac{M_2}{M_1}\right)^2 \quad (11)$$

In this model neutrino masses are proportional to the charged lepton masses, while in the Zee model the Majorana neutrinos are quadratic in the charged lepton masses. Note that one of eigenvalues of the neutrino mass matrix in Eq. (11) vanishes, since the 3×3 antisymmetric matrix $f_{\alpha\beta}$ has zero determinant. The same feature has been noticed in another model [8] with a doubly charged scalar. The higher loop correction can always be written in the form $g^T Y f$, where Y depends on the parameters of the model. Thus at any higher loop level one of the neutrinos remains massless. Note that although the charged lepton masses receive corrections at one loop by replacing the charged lepton fields in Fig.1 by the Dirac neutrino fields, this correction is $\delta m_l \simeq \frac{m_\nu^2}{m_l}$, which can safely be neglected.

Now let us study the phenomenological constraints on the parameters of the model. The physical charged scalar fields h_1 and h_2 can mediate the decays $\mu \longrightarrow e \nu_e \nu_\mu$, $\tau \longrightarrow e \nu_e \nu_\tau$, and $\tau \longrightarrow \mu \nu_\mu \nu_\tau$. The muon decay (see Fig. 2) can be described by the effective lagrangian:

$$\mathcal{L}_{eff} = -\frac{4G_F}{\sqrt{2}} [(1 + \delta_{Le\mu}) \bar{e}_L \gamma^\rho \nu_{eL} \bar{\nu}_\mu \gamma^\rho \mu_L + \delta_{RRe\mu} \bar{e}_R \gamma^\rho \nu_{eR} \bar{\nu}_{\mu R} \gamma^\rho \mu_R + \delta_{LRe\mu} \bar{e}_R \nu_{eL} \bar{\nu}_{\mu R} \mu_L], \quad (12)$$

where

$$\delta_{LLe\mu} = \frac{f_{e\mu}^2}{\sqrt{2}G_F\overline{M}^2}, \quad (13)$$

$$\delta_{RR_{e\mu}} = \frac{g_{e\mu}^2}{\sqrt{2}G_F\overline{M}^2}, \quad (14)$$

$$\delta_{LR_{e\mu}} = \frac{f_{e\mu}g_{e\mu}}{\sqrt{2}G_F\overline{M}^2}, \quad (15)$$

with:

$$\overline{M}^2 = \frac{M_1^2 M_2^2}{M_1^2 \cos^2 \theta + M_2^2 \sin^2 \theta} \quad (16)$$

For the $\tau \longrightarrow e\nu_e\nu_\tau$ and $\tau \longrightarrow \mu\nu_\mu\nu_\tau$ decays one has to substitute $\delta_{e\mu}$ by $\delta_{e\tau}$ and $\delta_{\mu\tau}$ respectively. The more stringent constraint on $f_{e\mu}$ is derived from universality of the μ decay which gives[9]

$$\frac{f_{e\mu}}{M} \leq 10^{-4} GeV^{-1}. \quad (17)$$

A useful constraint on $g_{e\mu}$ can be obtained from $e - \mu$ universality which gives the bound

$$\frac{g_{e\mu}f_{e\mu}}{\overline{M}^2} \leq 3 \times 10^{-6} GeV^{-2}. \quad (18)$$

The charged scalars h_1 and h_2 contribute to the $\mu \longrightarrow e\gamma$. A straightforward calculation gives:

$$\Gamma(\mu \longrightarrow e\gamma) \simeq \alpha \frac{1}{384\pi^4} \frac{m_\mu^5}{\overline{M}^4} |(ff^+)_{12} + (gg^+)_{12}|^2 \quad (19)$$

and by using the experimental upper limit $B(\mu \longrightarrow e\gamma) < 1.2 \times 10^{-11}$ we obtain

$$\frac{1}{\overline{M}^2} |(ff^+)_{12} + (gg^+)_{12}| < 2.8 \times 10^{-9} GeV^{-2}. \quad (20)$$

Another bound on the $(V + A)$ couplings $g_{\alpha\beta}$ can be driven from the constraint on the Michel parameter ζ . Using the experimental bounds from Ref. [10], we obtain

$$\frac{g_{e\mu}^2}{\overline{M}^2} < 3 \times 10^{-4} GeV^{-2} \quad (21)$$

Note that the decay $\mu \longrightarrow \bar{\nu}_e \nu_{\mu_R} e_R$ will be forbidden kinimatically in the seesaw case, since the mass of the right handed neutrino is much bigger than the mass of the muon. Note that for $M_{1,2} \simeq 10\text{TeV}$ and $f < 1, g < 1$ the above constraints are satisfied.

Now let us turn to the mixings and the masses relevant for neutrino oscillations, we assume for simplicity $g_{\alpha\beta} \simeq f_{\alpha\beta}$, such that the model has four remaining parameters. In this case the neutrino mass matrix is symmetric, and has the following form:

$$m_\nu \simeq m_0 \begin{bmatrix} f_{e\tau}^2 + f_{e\mu}^2 \left(\frac{m_\mu}{m_\tau}\right) & f_{e\tau} f_{\mu\tau} & -f_{e\mu} f_{\mu\tau} \left(\frac{m_\mu}{m_\tau}\right) \\ f_{e\tau} f_{\mu\tau} & f_{\mu\tau}^2 & f_{e\mu} f_{e\tau} \left(\frac{m_e}{m_\tau}\right) \\ -f_{e\mu} f_{\mu\tau} \left(\frac{m_\mu}{m_\tau}\right) & f_{e\mu} f_{e\tau} \left(\frac{m_e}{m_\tau}\right) & f_{\mu\tau}^2 \left(\frac{m_\mu}{m_\tau}\right) \end{bmatrix} \quad (22)$$

where m_0 corresponds to the scale of neutrino mass in this model, and it is given by:

$$m_0 = \frac{1}{64\pi^2} m_\tau \frac{4\kappa^2}{M_2^2 + 4\kappa^2}, \quad (23)$$

where in deriving the above relation we have assumed that the charged higgs h_2 is heavier than h_1 . For $\kappa \simeq M_2$, $m_0 \simeq 2\text{MeV}$, which implies that the parameters $f_{\alpha\beta} \leq 10^{-3}$. It is well known that a stable MeV neutrino mass can be problematic for cosmology [11]. Moreover primordial nucleosynthesis excludes a Dirac neutrino mass from .3 to 25MeV [12], which implies that $f_{\alpha\beta}$ should be less than one.

If there is no strong hierarchy between the Yukawa couplings $f_{\alpha\beta}$, then the matrix element $m_{\nu_{23}}$ can be safely neglected, and the eigenvalues of m_ν are

$$\begin{aligned} m_{\nu_1} &= 0 \\ m_{\nu_2} &\simeq m_0 \left(f_{\mu\tau}^2 + \frac{1}{2} \left(\frac{m_\mu}{m_\tau}\right) f_{e\mu}^2 - \sqrt{f_{e\tau}^4 + \frac{1}{4} \left(\frac{m_\mu}{m_\tau}\right)^2 f_{e\mu}^4} \right) \\ m_{\nu_3} &\simeq m_0 \left(f_{\mu\tau}^2 + \frac{1}{2} \left(\frac{m_\mu}{m_\tau}\right) f_{e\mu}^2 + \sqrt{f_{e\tau}^4 + \frac{1}{4} \left(\frac{m_\mu}{m_\tau}\right)^2 f_{e\mu}^4} \right) \end{aligned} \quad (24)$$

This means that m_{ν_3} is the scale relevant for atmospheric neutrino oscillations, which implies that $f_{\mu\tau} \sim f_{e\tau} \sim 10^{-4}$. However the explanation of the solar neutrino anomaly requires a fine

tuning between $f_{e\tau}$ and $f_{\mu\tau}$ such that $m_{\nu_2} \ll \Delta m_{atm}^2$. The matrix m_ν can be diagonalized (neglecting CP violation in the leptonic sector) by the following orthogonal transformation:

$$V = \begin{bmatrix} \frac{1}{2aC_1} & -\frac{a+b+\sqrt{a^2+b^2}}{c_2} & -\frac{a+b-\sqrt{a^2+b^2}}{c_3} \\ -\frac{1}{2aC_1} & \frac{a-b-\sqrt{a^2+b^2}}{c_2} & \frac{a-b+\sqrt{a^2+b^2}}{c_3} \\ \frac{1}{C_1} & \frac{c_2}{C_2} & \frac{c_3}{C_3} \end{bmatrix} \quad (25)$$

where :

$$a = \frac{f_{e\mu}}{2f_{\mu\tau}} \quad (26)$$

$$b = \frac{f_{\mu\tau}}{\epsilon f_{e\mu}} \quad (27)$$

$$c_1 = \frac{\sqrt{1+4a^2}}{a} \quad (28)$$

$$c_2 \simeq 2(a^2 + b^2 + a\sqrt{a^2 + b^2} + \frac{1}{4})^{\frac{1}{2}} \quad (29)$$

$$c_3 \simeq 2(a^2 + b^2 + b\sqrt{a^2 + b^2} + \frac{1}{4})^{\frac{1}{2}} \quad (30)$$

From the above mixing matrix one can see that the large mixing angle required for the atmospheric neutrino oscillations can not be satisfied simultaneously with the required smallness of the U_{e3} element. One could ask if the model can accomodate both the atmospheric and solar neutrino anomalies for $g_{\alpha\beta} \neq f_{\alpha\beta}$. However we have performed an analysis of all possible parameters $f_{\alpha\beta}$, $g_{\alpha\beta}$, and we found no solution that can fit both atmospheric and solar neutrino puzzles with $U_{e3} \leq 0.2$.

Now let us assume that lepton number is broken by the right handed neutrino mass term:

$$\mathcal{L}_{\Delta L=2} = \frac{1}{2} M_{R_{\alpha\beta}} N_{R_{\alpha\beta}}^T C N_{R_{\alpha\beta}} + h.c \quad (31)$$

The masses of the light neutrinos are obtained by diagonalizing the following mass matrix in the basis ν_L, N_R^c :

$$\begin{bmatrix} 0 & m_{\nu_D}^T \\ m_{\nu_D} & M_R \end{bmatrix} \quad (32)$$

where m_{ν_D} is the Dirac neutrino mass matrix induced at one loop as in Eq.(11). The seesaw approximation ($M_R \gg m_{\nu_D}$) implies

$$M_{\nu}^{(light)} \simeq -m_{\nu_D}^T M_R^{-1} m_{\nu_D} \quad (33)$$

We will assume that the Yukawa coupling constants $f_{\alpha\beta}$, $g_{\alpha\beta}$ are of order unity, and M_R, M_1 and M_2 in the TeV range. In this case the light Majorana neutrino masses are smaller than eV. Of course, one of the eigenvalues will vanish, and this requires the other masses to be in the atmospheric and solar neutrino ranges. This can be easily accommodated since the seesaw formula is very sensitive to any deviation of the coupling constants from unity, since it varies as the fourth power of the coupling constants.

The model can not predict the mixing angles since the parameters $f_{\alpha\beta}$ and $g_{\alpha\beta}$ are arbitrary, in contrast to the Zee model, in which the neutrino mass matrix has vanishing diagonal elements without any fine tuning or imposing extra symmetries. However, unlike the Dirac case, the seesaw neutrino mass matrix can accommodate the atmospheric and the solar anomalies. Choosing the Yukawa parameters $f_{\alpha\beta}, g_{\alpha\beta}$ to take the values:

$$\begin{aligned} g_{11} = g_{22} = g_{33} = g_{21} = g_{13} = g_{31} &= 1 \\ g_{23} = g_{32} &= 0 \\ g_{12} &\simeq -.23 \\ f_{23} &= 1 \end{aligned} \quad (34)$$

$$f_{12} = -f_{13} \simeq .5 \quad (35)$$

with the right handed neutrino masses $(M_{R_1}, M_{R_2}, M_{R_3}) \simeq (15.2, 3.3, 10) TeV$, leads to a bimaximal mixing with large angle MSW [13] effect as the solution to the solar neutrino puzzle. The numbers above suggest that one can easily choose a texture for the matrix $g_{\alpha\beta}$, that can fit both the atmospheric and solar neutrino data.

In summary we have proposed a model for generating small majorana neutrino masses at the TeV scale using the seesaw mechanism. The smallness of the neutrino masses in this model is not due to the largeness of the right handed neutrino mass, but to the smallness of the Dirac masses. The model may also have an interesting implication for cosmology[14].

1 Acknowledgment

One of the authors (S. N) would like to thank K. S. Babu for useful discussion, and very thankful to J. Schechter and M. Trodden for careful reading of the manuscript. S. M is very thankful to the Egyptian Cultural Bureau for the support.

References

- [1] Y. Fukuda et al. [Super-Kamiokande Collaboration], Phys. Rev. Lett. 81, 1562(1998); R. Davis et al, Proceeding of the Sixth International Workshop on neutrino Telescope, ed M. Baldoccalin Venezia, (1994); Hirata et al. Phys. Rev. D 44, 2241(1994); A. I. Abazov et al. Phys. Rev. Lett. 67, 332(1991); P. Anselman et al. Phys. Lett. B. 285, 376(1992); Phys. Lett. B 314, 445(1993).
- [2] M. Gell-man, P. Ramond and R. Slansky, in Supergravity, eds. P. van Nieuwenhuizen and D. Z. Freedman (North Holland 1979); T. Yanagida, in Proceedings of Workshop on Unified Theory and Baryon number in the Universe, eds. O. Sawada and A. Sugamoto (KEK 1979); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912(1980).
- [3] R. N. Mohapatra, hep-ph/9911272.
- [4] E. Ma and U. Sarkar, Phys. Rev. Lett 80, 5716(1998)

- [5] A. Zee, Phys. Lett. B 93, 339(1980); Phys. Lett. B 161, 141(1985); D. Chang and A. Zee, Phys. Rev. D 61, 071303(2001).
- [6] D. Chang and R. N. Mohapatra, Phys. Rev. Lett 58, 1600(1987); S. Bertolini and J. Liu, Nucl. Phys. B 297, 401(1987); G. C. Branco and C. Q. Geng, Phys. Rev. Lett 58, 969(1987); A. Masiero, D. V. Nanopoulos and A. I. Sanda, Phys. Rev. Lett 58, 1048(1986).
- [7] K. S. Babu and V. S. Mathur, Phys. Lett. B 196, 218; M. Fukugita and T. Yanagida, Phys. Rev. Lett 58, 1807(1987).
- [8] K. S. Babu, Phys. Lett. B 203, 132(1987).
- [9] A. Yu. Smirnov and M. Tanimoto, Phys. Rev. D 55, 1665(1997); g. C. McLaughlin and J. N. Ng, Phys. Lett. B 455, 224(1999); J.E.Kim and J. S Lee, hep-ph/9907452; E. Mitsuda and K. Sasaki, hep-ph/01033202; S. Kanemura, et al, hep-ph/0011357. A.Ghosal, Y. Koide and H. Fusaoka, hep-ph/0104104.
- [10] Particle Data Group, Eur. Phys. C15, 1(2000)
- [11] A. D. Dolgov and I. Z. Rothstein, Phys. Rev. Lett 71, 476(1993); G. G. Raffelt, hep-ph/9902271; A. D. Dolgov, S. Pastor and J. W. Valle, Phys. Lett. B 383(1996).
- [12] E. W. Kolb, M. S. Turner, A. Chakravorty and D. N. Schramm, Phys. Rev. Lett 67, 533(1991).
- [13] J. N. Bahcall, P.I. Krastev, and A. Yu. Smirnov, Phys. Rev. D60, 093001, 1999; M. C. Gonzalez-Garcia, M. Maltoni, C. Peña-Garay, and J. W. F. Valle, Phys. Rev. D63, 033005, 2001.
- [14] S. Nasri and M. Trodden, in progress (2001).

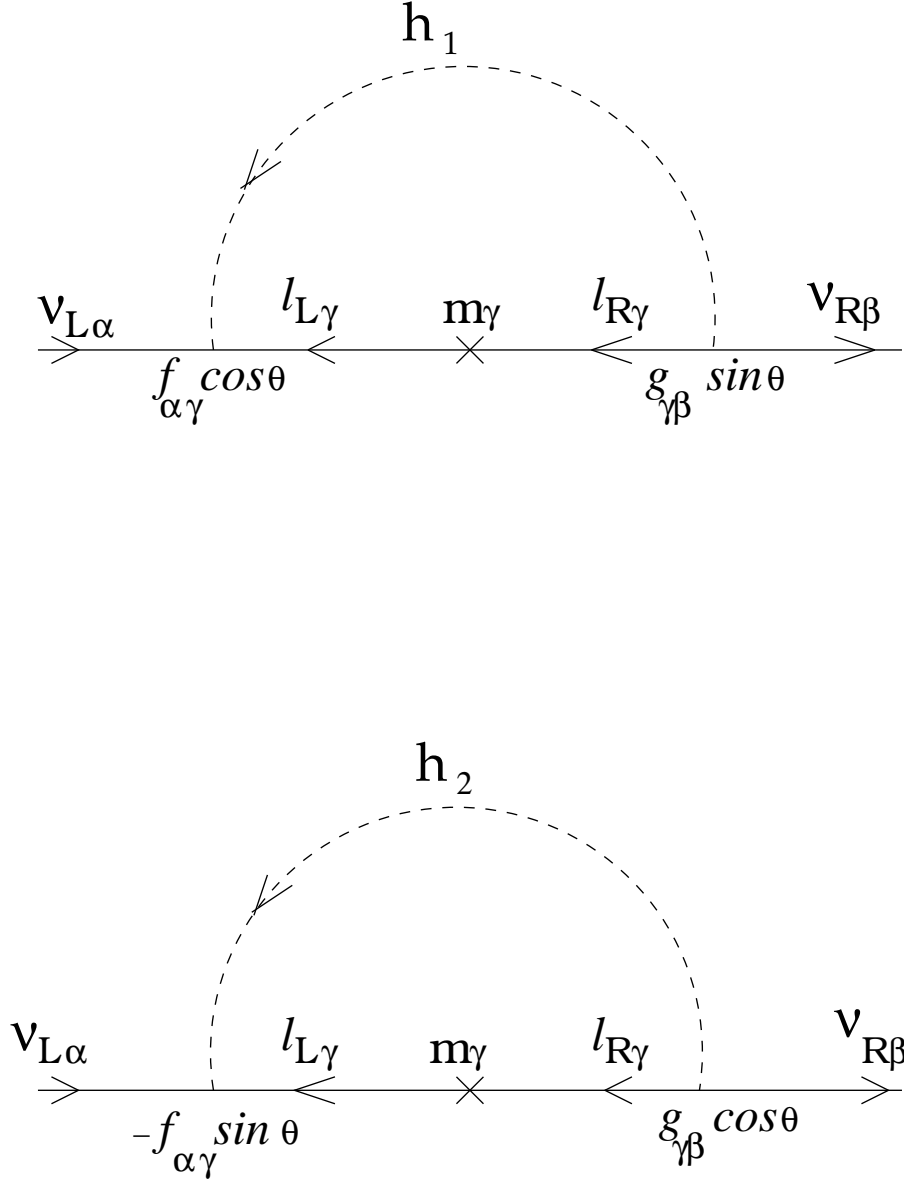


Figure 1: One loop diagram giving rise to Dirac neutrino masses

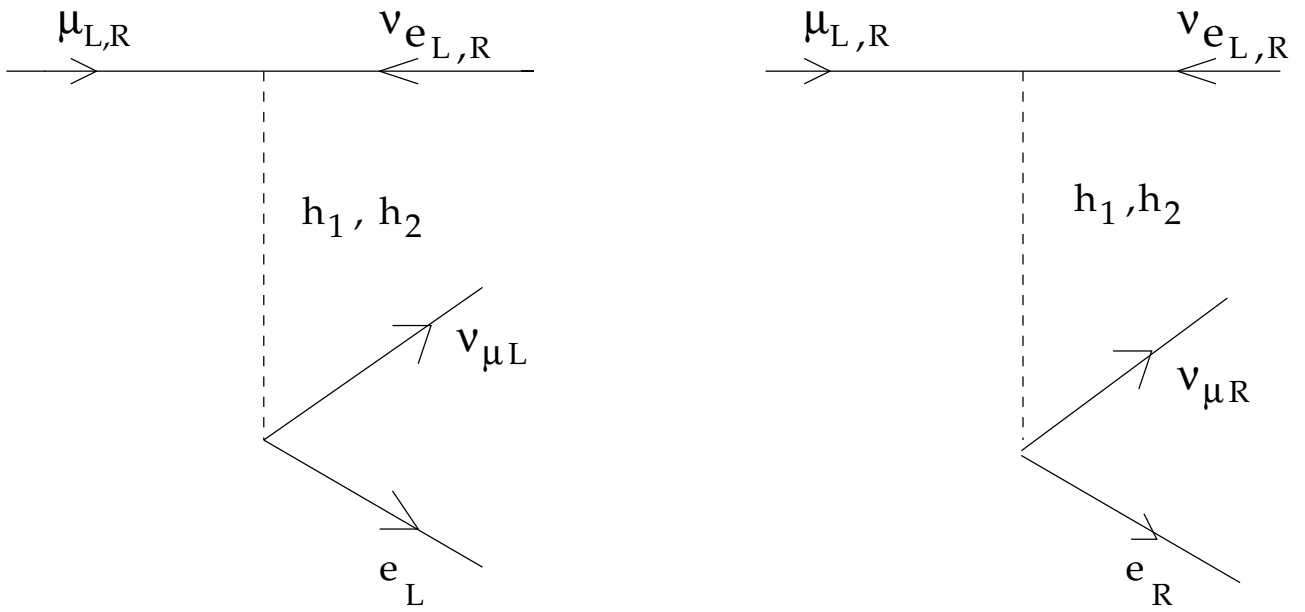


Figure 2: Muon decay via the exchange of the charged singlet fields